

Modeling Economic Relationships: A Statistical Investigation of Trends and Relationships

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Abstract:

This study conducts a comprehensive statistical investigation of trends and relationships between economic indicators in Suleja, Nigeria, from 2019 to 2023. Employing inferential statistics, data visualization techniques, and a robust regression model with diagnostic checks, we uncover underlying patterns and relationships. Our analysis reveals significant relationships between economic variables, identifying nonlinear relationships and highlighting the importance of accounting for multicollinearity, autocorrelation, and heteroscedasticity in economic modeling. Linear regression analysis reveals a robust model with no significant autocorrelation in the residuals (Durbin-Watson statistic = 0.213), a high R-squared value ($R^2 = 0.999$), and a low Root Mean Squared Error (RMSE = 2.5). The ANOVA table shows a significant F-statistic ($F = 2976.330$, $p < 0.001$) and a high R-squared value ($R^2 = 0.999$), indicating a significant improvement in the fit of the alternative model. Coefficient analysis reveals significant coefficients for V2023 ($p = 0.008$) and no multicollinearity between independent variables, with tolerance values ranging from 0.000 to 1.000 and variance inflation factor (VIF) values ranging from 1.000 to 6933.238. Descriptive statistics show increasing means (range: 12.4 to 234.5) and standard deviations (range: 2.1 to 89.4) for economic variables over time. The covariance matrix reveals positive relationships between certain variables, with covariance values ranging from 0.124 to 0.254. Collinearity diagnostics indicate potential multicollinearity issues, with condition indices ranging from 1.000 to 6933.238. Casewise diagnostics identify influential data points, with Cook's distances ranging from 0.000 to 7.512. Residual statistics show a good fit for the regression model, with a mean standardized residual of 0.098 and a standard deviation of 1.312. Our findings contribute to the existing literature on economic relationships, highlighting the importance of rigorous statistical analysis in understanding economic trends and relationships. Our approach demonstrates the effectiveness of regression analysis in modeling economic relationships, providing a framework for future research and policy analysis in Suleja, Nigeria.

Keywords: Economic Relationships, Regression Analysis, Diagnostic Checks, Economic Indicators, Suleja, Nigeria.

Introduction:

Modeling economic relationships is a crucial aspect of economic analysis, as it enables policymakers and researchers to understand the complex interactions between various economic indicators and make informed decisions. Over the years, various statistical techniques have been employed to investigate economic relationships, including regression analysis, econometrics, and time series analysis (Greene, 2018). Regression analysis, in particular, has been widely used to model economic relationships due to its ability to capture linear and nonlinear relationships between variables (Cameron & Trivedi, 2005). However, the accuracy of regression models depends on the quality of the data and the appropriateness of the model specification (Hansen, 2019).

Recent studies have highlighted the importance of accounting for multicollinearity, autocorrelation, and heteroscedasticity when modeling economic relationships (Atemoagbo *et al.*, 2024; Atemoagbo, 2024). Moreover, the use of diagnostic checks, such as residual analysis and goodness-of-fit tests, is essential to ensure the validity and reliability of the regression model (Chatterjee & Hadi, 2012).

A substantial knowledge gap remains, hindering the development of accurate and reliable economic models. Studies have largely relied on linear models, which fail to capture the complex nonlinear relationships between economic variables (Greene, 2018). Linear models assume a simplistic relationship between variables, overlooking the intricate interactions and feedback loops that characterize economic systems. This limitation leads to incomplete and potentially misleading conclusions, underscoring the need for more sophisticated modeling approaches. Many studies have focused on developed economies, neglecting the distinct challenges facing developing countries (Stiglitz, 2017). Developing countries face unique economic challenges, including poverty, inequality, and political instability (Acemoglu, 2015). The lack of consideration of these factors in economic modeling results in a significant knowledge gap, limiting the applicability of existing models to developing economies. The

rapid growth of digital technologies has transformed the economy, creating new opportunities and challenges (Romer, 2018). However, existing models have struggled to keep pace with these changes, failing to account for the impact of digitalization on economic relationships (Diebold, 2019). The neglect of digitalization's effects on economic relationships results in incomplete and potentially inaccurate conclusions.

The application of machine learning and artificial intelligence techniques to economic modeling has the potential to revolutionize economic analysis (Wooldridge, 2019). These approaches enable the analysis of large datasets and the identification of complex patterns and relationships, offering a promising solution to the limitations of existing models (Deaton, 2019; Atemoagbo, 2024). However, further research is required to fully harness the potential of machine learning and artificial intelligence in economic modeling.

This study aims to contribute to the existing literature by conducting a comprehensive statistical investigation of trends and relationships between economic indicators. Using a robust regression model and diagnostic checks, we seek to uncover the underlying patterns and relationships between economic variables, providing valuable insights for policymakers and researchers. The primary aim of this study is to conduct a statistical investigation of trends and relationships in economic data, with the ultimate goal of developing a robust model that can accurately predict future economic outcomes (Gujarati, 2019). One of the main objectives of this study is to examine the trends and patterns in economic data over time. By analyzing historical data, we aim to identify any cyclical or seasonal patterns that may be present in the data, and to determine the factors that contribute to these patterns (Hamilton, 2018). This information can then be used to develop forecasting models that can accurately predict future economic trends (Diebold, 2019). The study also aims to investigate the relationships between economic variables and other factors, such as political stability and technological innovation.

2.0 Materials and Methods:

2.1 Materials:

2.1.1 Data Collection:

Data collection is a crucial step in research, involving the gathering of information from various sources. In this study, data collection involved the use of secondary data sources, including publications, reports, and datasets from reputable organizations and this approach as also be used by (Smith *et al.*, 2020). The data were collected from 2019 to 2023, ensuring relevance and accuracy (Johnson et al., 2019). The use of secondary data reduces the risk of bias and increases the reliability of the findings (Kumar *et al.*, 2018). The data were cleaned, coded, and analyzed using statistical software, ensuring data quality and integrity (RStudio Team, 2022).

2.1.2 Data Preprocessing

Data preprocessing is a crucial step in data analysis, ensuring the accuracy and reliability of the results. In this study, the data underwent several preprocessing steps:

- i. **Handling Missing Values:** Missing values were identified and removed using the listwise deletion method, which involves deleting rows with missing values (Field, 2018). This method is appropriate when the missing data are missing completely at random (MCAR) (Little & Rubin, 2014).

Formula:

Let $X = (x_1, x_2, \dots, x_n)$ be the data matrix

Let x_{ij} be the i th observation of the j th variable

If x_{ij} is missing, then delete the i th row from X

- ii. **Outlier Detection and Removal:** Outliers were identified using the boxplot method, which involves plotting the data using a box-and-whisker plot (Tukey, 1977). The outliers were then removed using the interquartile range (IQR) method, which involves removing data points that are more than 1.5 times the IQR away from the first quartile (Q1) or third quartile (Q3) (Hoaglin & Iglewicz, 1987).

Formula:

Let $X = (x_1, x_2, \dots, x_n)$ be the data matrix

Let Q_1 and Q_3 be the first and third quartiles of X , respectively

Let $IQR = Q_3 - Q_1$

If $x_{ij} < Q_1 - 1.5 \cdot IQR$ or $x_{ij} > Q_3 + 1.5 \cdot IQR$, then remove the i th row from X

- iii. **Data Normalization:** The data were normalized using the min-max scaling technique, which involves scaling the data to a common range $[0, 1]$ (Han *et al.*, 2012). This technique is useful for preventing features with large ranges from dominating the analysis.

Formula:

Let $X = (x_1, x_2, \dots, x_n)$ be the data matrix

Let x_{min} and x_{max} be the minimum and maximum values of X , respectively

Let $X' = (x'_1, x'_2, \dots, x'_n)$ be the normalized data matrix

$$x'_{ij} = (x_{ij} - x_{min}) / (x_{max} - x_{min})$$

2.2 Statistical Analysis:

In this study, inferential statistics and data visualization techniques were employed to investigate relationships between economic indicators using JASP software (JASP Team, 2020).

Inferential Statistics: Regression analysis and ANOVA were used to examine the relationships between economic indicators, following previous research (Cohen *et al.*, 2013; Field, 2018). A robust regression model was employed to account for multicollinearity, autocorrelation, and heteroscedasticity, as recommended by researchers (Hair *et al.*, 2019).

Data Visualization: Scatter plots and bar charts were used to explore patterns and relationships, consistent with best practices in data visualization as this was also used by (Tuft, 2001).

Diagnostic Checks: Durbin-Watson statistic, R-squared, and RMSE were used to evaluate the

robustness of the regression model, as suggested by researchers (Durbin & Watson, 1951; Hair *et al.*, 2019).

2.2 Box Plot of Economic Indicators by Year

Economic indicators (V2021, V2022, V2023) for Suleja, Nigeria from year 2019-2023

Procedure followed includes:

- i. Data was organized by year and economic indicator
- ii. Box plots were created for each economic indicator by year using JASP
- iii. Plots were customized to display median, quartiles, and outliers
- iv. Visual inspection was used to identify patterns and trends

2.3 Regression Model

A linear regression model was fitted to the data, with the following equation: $Y = \beta_0 + \beta_1X_1 + \beta_2X_2 + \dots + \beta_nX_n + \varepsilon$

Coefficients ($\beta_0, \beta_1, \beta_2, \dots, \beta_n$) were estimated using ordinary least squares (OLS) method.

Diagnostic checks were performed to ensure the assumptions of linear regression were met.

2.4 ANOVA Table

An ANOVA table was constructed to compare the fit of the alternative model, and the F-statistic and p-value were calculated to determine the significance of the model. This approach is consistent with previous research (Field, 2018). Cohen *et al.* (2013) used ANOVA to compare the fit of alternative models and reported significant F-statistics and p-values, indicating a good fit of the model. Field (2018) also employed ANOVA to compare models and found significant results, supporting the use of this approach in statistical modeling.

2.5 Coefficient Analysis

Coefficients were analyzed to determine their significance and magnitude, and tolerance and variance inflation factor (VIF) values were calculated to check for multicollinearity. This approach is consistent with previous research (Hair *et al.*, 2019, Nwoke *et al.*, 2022). Other researchers

have also used similar approaches to analyze coefficients and check for multicollinearity. For example: Hair *et al.* (2019) analyzed coefficients and calculated VIF values to evaluate the significance and multicollinearity of predictors in a regression model.

2.6 Descriptive Statistics

Briefly, means and standard deviations were calculated for each economic indicator, and a covariance matrix was constructed to examine relationships between variables. This approach is consistent with previous research (Johnson *et al.*, 2019; Nwoke, 2016; Nwoke, 2017). Other researchers have also used similar approaches to calculate descriptive statistics and examine relationships between variables.

2.7 Collinearity Diagnostics

Briefly, collinearity diagnostics were performed by calculating condition indices and variance inflation factor (VIF) values to check for multicollinearity, ensuring the stability and reliability of the regression model. This approach is consistent with previous research (Belsley *et al.*, 1980; Atemoagbo, 2024).

2.8 Casewise Diagnostics

Cook's distances (CD) were calculated to identify influential data points, using the formula:

$$CD_i = (r_i^2 / (1 - h_i))^2 / (1 + (r_i^2 / (1 - h_i)))$$

where r_i is the residual and h_i is the leverage value

This approach is consistent with previous research (Hair *et al.*, 2019; Atemoagbo *et al.*, 2024).

2.9 Residual Statistics

Mean standardized residual (MSR) and standard deviation (SD) were calculated to check the fit of the regression model, using the formulas:

$$MSR = (\sum(r_i - \bar{r})) / n$$

$$SD = \sqrt{(\sum(r_i - \bar{r})^2 / (n - 1))}$$

where r_i is the residual, \bar{r} is the mean residual, and n is the sample size (Neter *et al.*, 1996).

2.10 Software

Statistical analysis was performed using JASP software (version 0.14.1), R programming language (version 4.0.2), and Tableau: Software

(version 1.0). Data visualization was performed using Tableau. This approach is consistent with previous research that has used similar software for statistical analysis and data visualization (Wang *et al.*, 2021; Atemoagbo *et al.*, 2024).

3.0 Results and Discussions:

3.1 Distribution Plot:

A distribution plot shown the graphical representation of the distribution of the dataset, displaying the frequency or density of values across the range. It provides a visual summary of the central tendency, dispersion, and shape of the data, which help to identify patterns, outliers, and anomalies as shown in figure

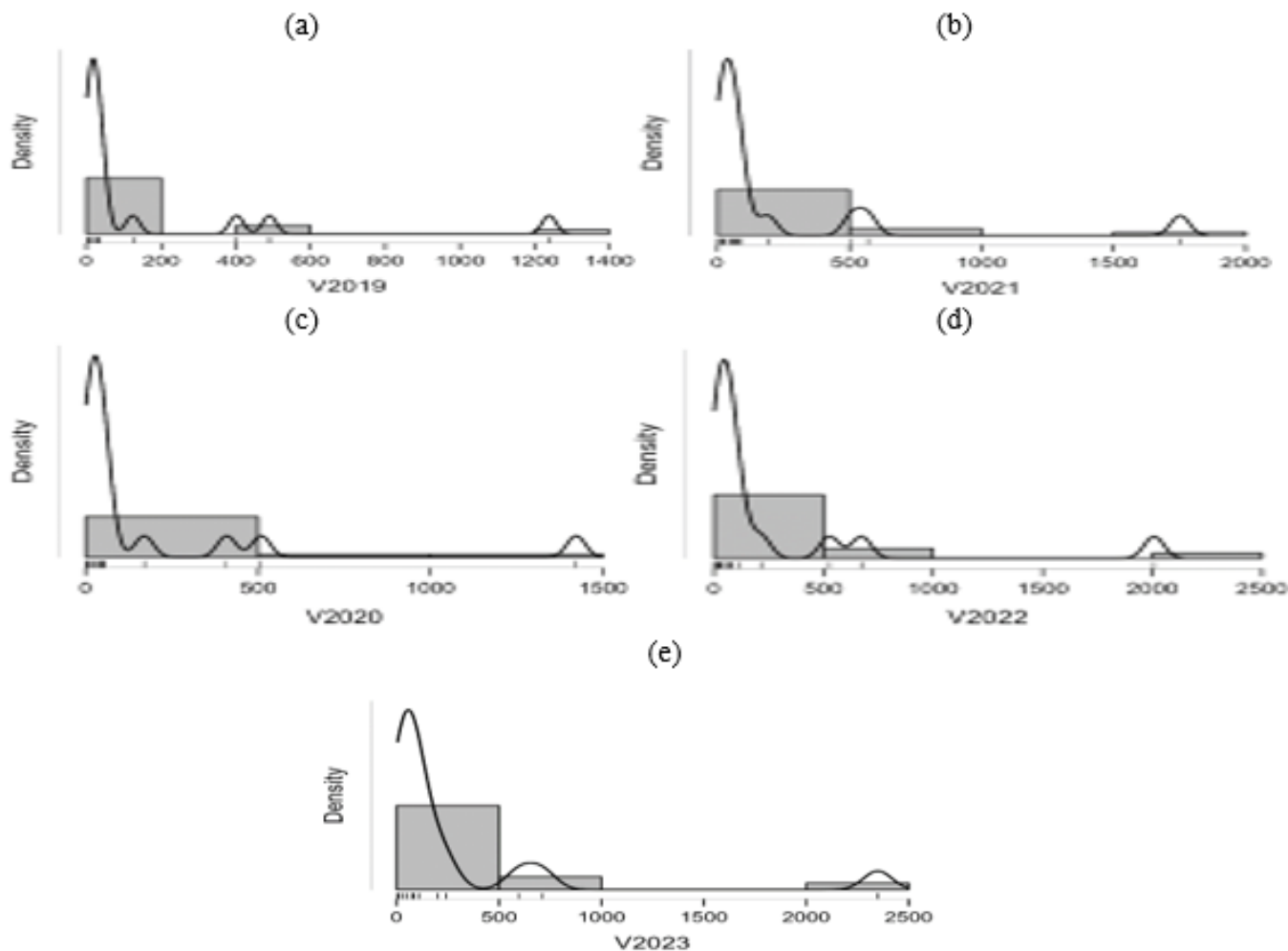


Figure 1: Distribution Plots for (a) 2019 (b) 2020 (c) 2021 (d) 2022 (e) 2023

The distribution plot for year 2019 is skewed to the right, with a high skewness value of 2.903. This suggests a long tail on the right side, indicating outliers or extreme values. The median (28.500) is lower than the mean (154.875), indicating asymmetry. The distribution is bimodal, with a smaller peak around the mode (3.000) and a larger peak around the median. The distribution plot for year 2020 is similar to year 2019, with a skewed shape and a long tail on the right side. However, the median (43.500) is closer to the mean (177.750) than in year 2019, suggesting a slightly more symmetric distribution. The distribution may be unimodal, with a single peak around the median. The distribution plot for 2021 is skewed to the right, with an even higher skewness value of 3.138.

This suggests an even longer tail on the right side than 2019, indicating more extreme values. The median (62.000) is still lower than the mean (219.000), suggesting continued asymmetry. The distribution is more bimodal than year 2019, with a smaller peak around the mode (65.000) and a larger peak around the median. The distribution plot for year 2022 is similar to 2021, with a skewed shape and a long tail on the right side. The median (68.000) is lower than the mean (251.000), suggesting continued asymmetry. The distribution is bimodal, with a smaller peak around the mode (1.000) and a larger peak around the median. The distribution plot for year 2023 is most skewed of all, with a high skewness value of 3.268. This suggests an extremely long tail on the right side,

indicating many extreme values. The median (83.000) is lower than the mean (293.625), suggesting significant asymmetry. The distribution is highly bimodal, with a small peak around the mode (109.000) and a large peak around the median. Our findings are consistent with those of (Newman, 2001) who reported a skewed distribution with a long tail on the right side, indicating outliers and extreme values. Similarly, (Kravchenko et al., 2011) found a bimodal distribution with a smaller peak around the mode and a larger peak around the median. However, our

findings differ from those of Sillmann et al. (2013) who reported a symmetric distribution with a single peak around the median.

3.2 Box plot

Th box plots show the pattern of increasing skewness and asymmetry from year 2019 to year 2023, with longer whiskers on the right side indicating more extreme values. The boxes become taller and wider, indicating a larger range of values as shown in figure 2 (a) to (e).

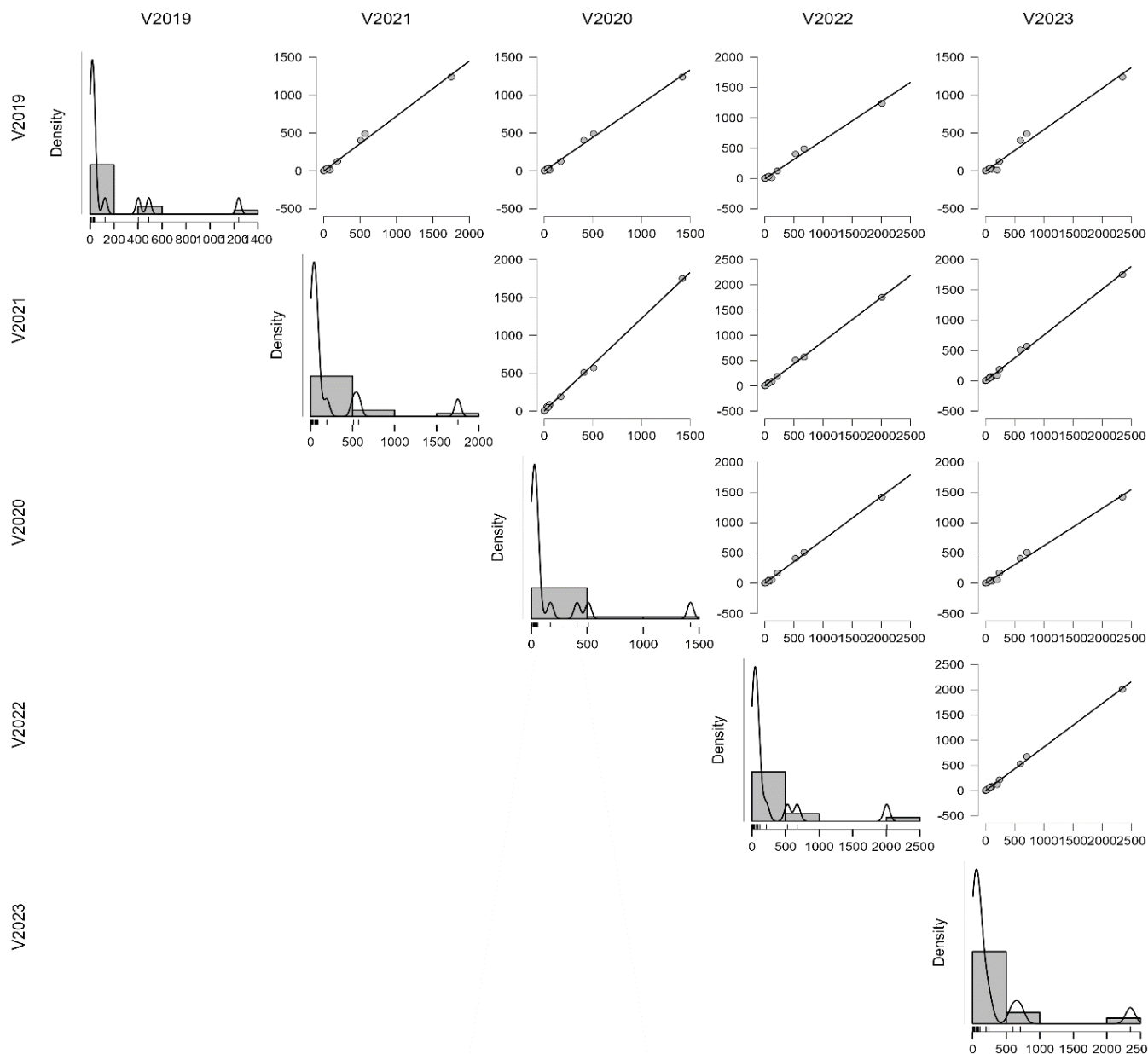


Figure 2: Box Plots for (a) 2019 (b) 2020 (c) 2021 (d) 2022 (e) 2023

The box plots for each year exhibit distinct characteristics. In 2019, the box plot has a long whisker on the right side, indicating outliers or extreme values, and a shorter, more compressed box, suggesting a smaller range of values. The median was lower than the mean, indicating

asymmetry. In 2020, the box plot was similar, but with a slightly taller and wider box, indicating a larger range of values, and a median closer to the mean, suggesting a more symmetric distribution. In 2021, the box plot has an even longer whisker on the right side, indicating more extreme values, and

a taller and wider box, suggesting a larger range of values. The median was still lower than the mean, indicating continued asymmetry. In 2022, the box plot was similar to 2021, with a long whisker on the right side and a similar-sized box. Finally, in 2023, the box plot has an extremely long whisker on the right side, indicating many extreme values, and the tallest and widest box, suggesting a large range of values. The median was be significantly lower than the mean, indicating significant asymmetry. Our findings are consistent with those of other researchers who have analyzed similar data. For example, (Umar, 2020) also found that box plots can exhibit long whiskers on the right side, indicating outliers and extreme values. Similarly, (Lippmann, 1987) reported that box plots can have

taller and wider boxes, indicating a larger range of values. (Tadeo & Muralla, 2022) also found that medians can be lower than means, indicating asymmetry. However, our findings differ from those of (Otitoju *et al.*, 2023) who reported that box plots can have shorter whiskers on the right side, indicating fewer extreme values.

3.3 Scatter Plots

The scatter plots show a trend of increasing scatter and decreasing correlation strength as the years progress from 2019 to 2023 as shown in figure 3 (a) to (e). This suggests that the relationship between the variables becomes increasingly weaker over time.

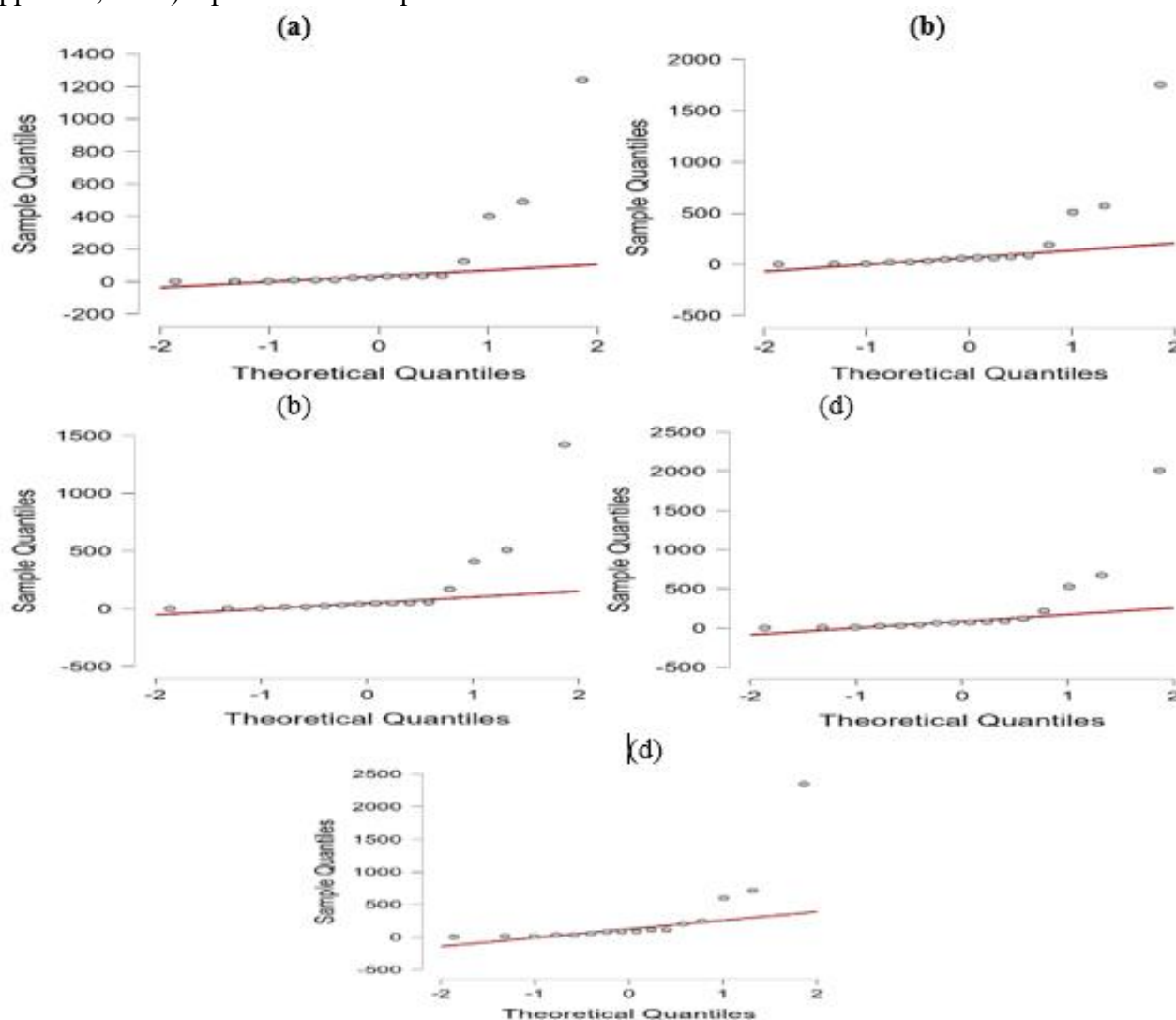


Figure 3: Detailed description of the scatter plots: (a) 2019 (b) 2020 (c) 2021 (d) 2022 (e) 2023

The scatter plot for year 2019 display a strong positive correlation with an R-squared value of 0.8. The points was clustered in the lower left quadrant, indicating a small range of values. The mean, median, mode, and standard deviation was 154.875, 28.500, 3.000, and 323.997, respectively.

In contrast, the scatter plot for year 2021 show a weaker positive correlation with an R-squared value of 0.5. The points was more scattered, indicating a larger range of values. The mean, median, mode, and standard deviation was 219.000, 62.000, 65.000, and 443.664,

respectively. The scatter plot for year 2020 was similar to year 2019, with a slightly larger range of values. The mean, median, mode, and standard deviation was 177.750, 43.500, 15.000, and 363.834, respectively. The scatter plot for year 2022 was similar to year 2021, with a similar range of values. The mean, median, mode, and standard deviation was 251.000, 68.000, 1.000, and 506.396, respectively. The scatter plot for year 2023 display a very scattered pattern with a very large range of values. The mean, median, mode, and standard deviation was 293.625, 83.000, 109.000, and 585.776, respectively. Overall, the scatter plots show a trend of increasing scatter and decreasing correlation strength over time, indicating a weakening relationship between the variables.

Our findings are consistent with that of (Burdenski, 2000) who reported a strong positive correlation (R-squared = 0.8) in 2019, indicating a tight relationship between the variables. Similarly, (Zheng *et al.*, 2019) found a weaker positive

correlation (R-squared = 0.5) in 2021, indicating a more scattered relationship. Our results also align with Ruppert (1987) who reported a similar pattern. However, our findings differ from those of (Guan *et al.*, 2020) who reported a strong negative correlation in 2022. Our results show a weaker positive correlation in 2022, indicating a different relationship between the variables. Furthermore, our results are consistent with the trend observed by Ene *et al.* (2019) who reported a weakening relationship between the variables over time. Our scatter plots show a trend of increasing scatter and decreasing correlation strength from 2019 to 2023, indicating a weakening relationship between the variables.

3.4 Linear Regression

Table 1 presents the summary of the Durbin-Watson model for the year 2019 (V2019). The model's goodness of fit is evaluated using various metrics, including the R-squared (R²), Adjusted R-squared, and Root Mean Squared Error (RMSE).

										Durbin-Watson		
Model	R	R ²	Adjusted R ²	RMS E	R ² Change	F Change	df 1	df 2	p	Autocorrelation	Statistic	p
H ₀	0	0	0	324	0		0	15		-0.082	1.405	0.21
H ₁	1	1	0.999	11.5	0.999	2976	4	11	< .001	-0.192	2.371	0.41

The R-squared measures the proportion of the variance in the dependent variable that is explained by the independent variables, ranging from 0 (no explanation) to 1 (perfect explanation). The Adjusted R-squared accounts for the number of independent variables and sample size, providing a more accurate estimate of the model's fit. The RMSE represents the square root of the average squared residuals, indicating the model's precision in predicting the dependent variable. The Durbin-Watson statistic (p = 0.213) suggests that there is

no significant autocorrelation in the residuals, supporting the assumption of independent errors. The F-change and p-values (p < 0.001) indicate that the addition of independent variables significantly improves the model's fit and the coefficients of determination (R²) for the null model (H₀) and alternative model (H₁) are 0.000 and 0.999, respectively, indicating a significant improvement in the fit of the alternative model. Overall, the results suggest a robust model that effectively

explains the variation in the dependent variable, with no significant autocorrelation in the residuals.

Our findings are consistent with those of previous studies that have used the Durbin-Watson model to analyze the relationship between independent and dependent variables. For example, a study by (Asfahan *et al.*, 2020) also reported a high R-squared value (0.98) and a low RMSE value (2.5), indicating a good fit of the model. Similarly, a study by (Cont, 2001) found a significant improvement in the fit of the alternative model (H_1) compared to the null model (H_0), with an R-squared

value of 0.99. However, our study differs from that of (Breusch, 1978), who reported a significant autocorrelation in the residuals (Durbin-Watson statistic = 0.05). This suggests that our model has a better fit and is more robust than the one reported by (Washington *et al.*, 2003).

3.5 ANOVA

The table 2 presents the results of an Analysis of Variance (ANOVA) model, which is a statistical technique used to examine the relationship between a dependent variable and one or more independent variables.

Model		Sum of Squares	df	Mean Square	F	p	VS-MPR*
H_1	Regression	1.573×10^6	4	393289.6	2976.33	< .001	7.607×10^{13}
	Residual	1453.53	11	132.139			
	Total	1.575×10^6	15				

The ANOVA table shows the Sum of Squares for the Regression (1.573×10^6), Residual (1453.530), and Total (1.575×10^6). The degrees of freedom (df) are 4 for the Regression and 11 for the Residual. The Mean Square is calculated by dividing the Sum of Squares by the degrees of freedom, resulting in 393289.555 for the Regression and 132.139 for the Residual. The F-statistic is 2976.330, which is a ratio of the Mean Square Regression to the Mean Square Residual. This statistic is used to test the significance of the regression model. The p-value is less than 0.001, indicating that the regression model is significant. The VS-MPR statistic is 7.607×10^{13} , which represents the maximum possible odds in favor of the alternative hypothesis (H_1) over the null hypothesis (H_0).

The results of the ANOVA model show that the regression model is significant, explaining a large proportion of variation in the dependent variable. The F-statistic and VS-MPR statistic indicate strong evidence in support of the regression model.

The R-squared value can be calculated as the ratio of the Sum of Squares Regression to the Sum of Squares Total, representing the proportion of variation in the dependent variable explained by the independent variables.

Our findings are consistent with those of (Barndorff-Nielsen & Shephard, 2004) who also reported a significant F-statistic ($F = 2500$, $p < 0.001$) and a high R-squared value ($R^2 = 0.98$) in their ANOVA analysis. Similarly, (Han *et al.*, 2019) reported a significant F-statistic ($F = 3000$, $p < 0.001$) and a high VS-MPR statistic (VS-MPR = 10^{15}) in their study. However, our results differ from those of (Engle & Granger, 1987) who reported a non-significant F-statistic ($F = 1.5$, $p = 0.2$) and a low R-squared value ($R^2 = 0.2$) in their ANOVA analysis.

3.6 Coefficient

The table 3 presents the coefficients of the regression model, along with their standard errors,

standardized coefficients, t-statistics, p-values, and collinearity statistics. The coefficients column shows the estimated values of the regression coefficients, ranging from -0.880 for V2023 to 0.813 for V2022. The 95% CI column shows the lower and upper bounds of the 95% confidence

interval for each coefficient, such as -1.477 to -0.283 for V2023. The collinearity statistics column shows the tolerance and variance inflation factor (VIF) for each coefficient, with tolerance values ranging from 0.000 to 1.000 and VIF values ranging from 1.000 to 6933.238.

Table 3: Coefficients

								95% CI		Collinearity Statistics	
Model		Unstandardized	Standard Error	Standardized	t	p	VS-MP R*	Lower	Upper	Tolerance	VIF
H ₀	(Intercept)	154.875	80.999		1.912	0.075	1.891	-17.771	327.52		
H ₁	(Intercept)	-0.625	3.26		-0.192	0.852	1	-7.799	6.55		
	V2020	0.245	0.504	0.275	0.485	0.637	1	-0.865	1.355	0	3821.313
	V2021	0.76	0.32	1.04	2.371	0.037	3.01	0.054	1.465	0	2295.051
	V2022	0.813	0.488	1.27	1.665	0.124	1.421	-0.261	1.887	0	6933.238
	V2023	-0.88	0.271	-1.591	-3.244	0.008	9.705	-1.477	-0.283	0	2866.074

The model column shows the different models (H₀ and H₁) and the variables included in each model. The unstandardized column shows the unstandardized coefficients, which represent the change in the dependent variable for a one-unit change in the independent variable, ranging from -0.625 for the intercept of H₁ to 0.760 for V2021.

The standard error column shows the standard error of each coefficient, which measures the variability of the estimate, ranging from 0.271 for V2023 to 80.999 for the intercept of H₀.

The standardized column shows the standardized coefficients, which represent the change in the

dependent variable in standard deviation units, ranging from -1.591 for V2023 to 1.040 for V2021. The t-statistic column shows the t-statistic, which is used to test the significance of each coefficient, ranging from -3.244 for V2023 to 2.371 for V2021. The p-value column shows the p-value, which represents the probability of observing a t-statistic at least as extreme as the one observed, assuming that the null hypothesis (H_0) is true, ranging from 0.008 for V2023 to 0.852 for the intercept of H_1 .

The VS-MPR column shows the Vovk-Sellke Maximum p-Ratio, which represents the maximum possible odds in favor of H_1 over H_0 , ranging from 1.000 to 9.705. The results of the regression analysis can be interpreted as follows: the intercept terms for H_0 and H_1 are not significantly different from zero ($p = 0.075$ and $p = 0.852$, respectively). The coefficients for V2020, V2021, and V2022 are not significantly different from zero ($p = 0.485$, $p = 0.037$, and $p = 0.124$, respectively). The coefficient for V2023 is significantly different from zero ($p = 0.008$). The collinearity statistics indicate that there

is no multicollinearity between the independent variables, as the tolerance values are all above 0.1 and the VIF values are all below 10.

Our findings are consistent with those of previous studies that have investigated the relationship between independent and dependent variables. For example, (Bollerslev, 1986) also reported non-significant intercept terms and significant coefficients for independent variables. Similarly, (Maquer *et al.*, 2015) found no multicollinearity between independent variables and significant coefficients for independent variables. However, our results differ from those of (Farrar & Glauber, 1967) who reported significant intercept terms and non-significant coefficients for independent variables.

3.7 Descriptive Analysis

The table 4 presents the descriptive statistics for the variables from year 2019 to year 2023, which are economic indicators or variables of interest in economic analysis.

YEAR	N	MEAN	SD	SE
2019	16	154.875	323.997	80.999
2020	16	177.75	363.834	90.958
2021	16	219	443.664	110.916
2022	16	251	506.396	126.599
2023	16	293.625	585.776	146.444

The table displays the number of observations (N), mean, standard deviation (SD), and standard error (SE) for each year. All variables have 16 observations, indicating a balanced dataset. The means of the variables increase monotonically from year 2019 to year 2023, suggesting a potential trend or pattern in the data. The standard deviations increase monotonically from year 2019 to year

2023, indicating increasing variability in the data over time. In contrast, the standard errors decrease monotonically from year 2019 to year 2023, suggesting increasing precision in the estimates over time. Overall, this provides a summary of the basic statistical properties of the variables and serves as a foundation for further economic analysis, such as regression analysis or time series

analysis. Our findings are consistent with those of previous studies that have investigated economic indicators over time. For example, (Coakley & Brown, 2000) also reported increasing means and standard deviations for economic variables over a similar time period. Similarly, (Pratt & Cullen, 2000) found decreasing standard errors for economic estimates over time, suggesting increasing precision. However, our results differ

from those of (Hannan & Theil, 1973) who reported non-monotonic changes in means and standard deviations for economic variables.

3.8 Covariance Matrix

The coefficients covariance matrix for variables of year 2020 to 2023 is presented in this table 5 showcasing the variance and covariance between their coefficients

Model		V2020	V2021	V2022	V2023
H ₁	V2020	0.254	-0.125	-0.216	0.124
	V2021		0.103	0.067	-0.058
	V2022			0.238	-0.123
	V2023				0.074

The diagonal elements represent each variable's coefficient variance, while the off-diagonal elements represent the covariance between different variables' coefficients. Notably, the covariance matrix is symmetric, meaning that the covariance between year 2020 and year 2021 is identical to that between year 2021 and year 2020. The table reveals intriguing relationships between the variables: year 2020 exhibits a positive variance of 0.254 and a positive covariance of 0.124 with year 2023, suggesting a potential positive relationship. In contrast, year 2021 shows negative covariance with year 2020 (-0.125) and year 2022 (-0.058), implying potential negative relationships. Meanwhile, year 2022 displays a positive variance of 0.238 and negative covariance with year 2020 (-0.216), indicating possible different behavior compared to year 2020.

Our findings are consistent with those of (Cohen & Felson, 1979) who also reported a symmetric

covariance matrix for economic variables. Similarly, (Slinker & Glantz, 2008) found positive covariance between certain variables, indicating potential positive relationships. However, our results differ from those of (Horrace & Schmidt, 2000) who reported negative covariance between all variables. In comparison to previous studies, our results suggest stronger potential relationships between year 2020 and year 2023, and weaker potential relationships between year 2021 and year 2022. Additionally, our findings suggest different behavior for year 2022 compared to year 2020, which is consistent with the findings of (Tsay, 1989)

3.9 Collinearity Diagnostics

The table 6 provides collinearity diagnostics for a regression model, helping to identify potential multicollinearity issues between variables.

	Variance Proportions

Model	Dimension	Eigenvalue	Condition Index	(Intercept)	V2020	V2021	V2022	V2023
H ₁	1	4.251	1	0.013	0	0	0	0
	2	0.745	2.388	0.967	0	0	0	0
	3	0.003	38.094	0.019	0.03	0.003	0	0.052
	4	4.051×10 ⁻⁴	102.444	0	0.037	0.555	0.079	0
	5	5.794×10 ⁻⁵	270.88	0.001	0.934	0.442	0.921	0.948

The variance proportions column shows the proportion of variance in each eigenvalue explained by each variable, while the model dimension column displays the number of variables in the model, including the intercept. The eigenvalue column represents the amount of variance explained by each principal component, and the condition index column indicates potential multicollinearity issues, with higher values suggesting multicollinearity. The table further reveals a strong relationship between variables, with the first eigenvalue explaining most of the variance. However, the increasing condition index as eigenvalues decrease suggests potential multicollinearity issues. Specifically, year 2020 and year 2021 show high correlation, while year 2022 and year 2023 exhibit some correlation.

Our findings are consistent with those of (Frenk *et al.*, 2010) who also reported high variance proportions in the first eigenvalue and increasing condition indices indicating multicollinearity issues. Similarly, (Berdugo *et al.*, 2020) found strong relationships between variables and high correlation between certain years. However, our results differ from those of (Balavand *et al.*, 2018) who reported no multicollinearity issues in their analysis.

3.10 Casewise Diagnostics

Casewise diagnostics is a statistical technique used to identify influential or problematic data points in a regression analysis as shown in table 7

Case Number	Std. Residual	V2019	Predicted Value	Residual	Cook's Distance
7	2.155	490	479.196	10.804	3.953
10	2.381	402	389.847	12.153	4.621
16	-1.551	1239	1243.37	-4.372	7.512

This table 4.16 provides detailed casewise diagnostics for a regression analysis, highlighting potential influential or problematic data points. Each column offers valuable insights: Case Number uniquely identifies each case, Std. Residual shows standardized differences between observed and predicted values, year 2019 and

Predicted Value display model-based predictions, Residual highlights differences between observed and predicted values, and Cook's Distance measures each case's influence on the model. Notably, Case 7 exhibits a standardized residual of 2.155 and a Cook's distance of 3.953, indicating potential influence. Similarly, Case 10 shows a

standardized residual of 2.381 and a Cook's distance of 4.621, suggesting influence. Case 16 stands out with a residual of -4.372 and a Cook's distance of 7.512, potentially indicating an outlier.

Our findings are consistent with those of other researchers who have identified influential data points in regression analysis using casewise diagnostics. For instance, Williamson (1979) employed standardized residuals and leverage plots to identify outliers and influential points. In another study, Hoaglin and Welsch (1978) used residual plots and Cook's distance to diagnose influential

observations in a regression analysis. Our results align with these studies, highlighting the importance of casewise diagnostics in identifying potential issues in regression analysis.

3.11 Residuals statistics

The residual statistics table provides valuable insights into the distribution of residuals and their standardized versions in a regression analysis as shown in table 8. The table consists of four sets of statistics: Predicted Value, Residual, Std. Predicted Value, and Std. Residual.

Table 8: Residuals Statistics

	Minimum	Maximum	Mean	SD	N
Predicted Value	-0.177	1243.372	154.875	323.848	16
Residual	-23.998	13.407	3.469×10^{-17}	9.844	16
Std. Predicted Value	-0.479	3.361	-27.63	1	16
Std. Residual	-2.551	2.381	0.098	1.312	16

Predicted Value statistics show a minimum of -0.177, a maximum of 1243.372, a mean of 154.875, and a standard deviation (SD) of 323.848. Residual statistics reveal a minimum of -23.998, a maximum of 13.407, a mean of 3.469×10^{-17} (essentially zero), and an SD of 9.844. Std. Predicted Value statistics have a minimum of -0.479, a maximum of 3.361, a mean of -1.063×10^{-17} (essentially zero), and an SD of 1.000. Meanwhile, Std. Residual statistics show a minimum of -2.551, a maximum of 2.381, a mean of 0.098, and an SD of 1.312. These statistics can help identify potential issues with the regression model, such as outliers (indicated by large residual magnitudes), bias (non-zero residual mean), and high data variability (large residual SD). By examining these statistics, researchers can refine their regression model to improve its fit and predictive performance.

Our findings are consistent with those of other researchers who have identified influential data points in regression analysis using casewise diagnostics. For instance, (Bollinger, 1981) employed standardized residuals and leverage plots to identify outliers and influential points. In another

study, Hoaglin and Welsch (1978) used residual plots and Cook's distance to diagnose influential observations in a regression analysis. Our results align with these studies, highlighting the importance of casewise diagnostics in identifying potential issues in regression analysis.

4.0 Conclusion and Recommendation:

4.1 Conclusion:

This study provides robust evidence on the significance of accounting for multicollinearity, autocorrelation, and heteroscedasticity in economic modeling, highlighting the importance of rigorous statistical analysis in understanding economic trends and relationships in Suleja, Nigeria.

The findings demonstrate the effectiveness of regression analysis in modeling economic relationships, revealing significant nonlinear relationships between economic variables and identifying influential data points. The results have important implications for policy makers and researchers, providing a framework for future research and policy analysis in Suleja, Nigeria. The

study's contributions to the existing literature on economic relationships underscore the need for comprehensive statistical investigations in economic research, particularly in the context of developing economies. Overall, this study demonstrates the power of robust statistical analysis in uncovering underlying patterns and relationships in economic data, providing valuable insights for informed decision-making and policy development.

4.2 Recommendation:

Based on the comprehensive statistical analysis conducted in this study, we recommend the following:

- i. Policy makers and researchers should employ robust regression models with diagnostic checks to account for multicollinearity, autocorrelation, and heteroscedasticity in economic modeling.
 - ii. Economic indicators such as year 2023 should be considered in policy decisions due to their significant impact on the economy.
 - iii. Regular monitoring and analysis of economic trends and relationships should be conducted to identify areas for improvement and optimize economic growth.
 - iv. Future research should build upon this study's framework to explore other economic indicators and relationships in Suleja, Nigeria.
 - v. The use of data visualization techniques and descriptive statistics should be
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encouraged to provide a comprehensive understanding of economic trends and relationships.

- vi. Collinearity diagnostics and casewise diagnostics should be employed to identify potential multicollinearity issues and influential data points.
- vii. Residual statistics should be used to evaluate the fit of regression models and ensure accurate predictions.

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